3-6 Videos Guide

3-6a

- The directional derivative of $f(x, y)$ in the direction of $\mathbf{u} = \langle a, b \rangle$, a unit vector:
  $D_{\mathbf{u}} f(x, y) = f_x(x, y)a + f_y(x, y)b$
- In $\mathbb{R}^3$, for $f(x, y, z)$ and $\mathbf{u} = \langle a, b, c \rangle$, a unit vector:
  $D_{\mathbf{u}} f(x, y, z) = f_x(x, y, z)a + f_y(x, y, z)b + f_z(x, y, z)c$

3-6b

- The gradient vector
  $\nabla f = \langle f_x, f_y \rangle$ or $\nabla f = \langle f_x, f_y, f_z \rangle$
- The del operator ∇
- Dot product representation of the directional derivative
  $D_{\mathbf{u}} f = \nabla f \cdot \mathbf{u}$

Exercises:

- Find the directional derivative of $f(x, y) = xy^3 - x^2$ at the point $(1, 2)$ in the direction $\theta = \frac{\pi}{3}$.

3-6c

- Find the directional derivative of the function $f(x, y, z) = xy^2 \tan^{-1} z$ at the point $(2, 1, 1)$ in the direction $\mathbf{v} = \langle 1, 1, 1 \rangle$.
- Characteristics of the gradient vector
  - $\nabla f(a, b)$ points in the direction of maximum change of f
  - The maximum rate of change of f at (a, b) is $|\nabla f(a, b)|$

3-6d

- Tangent plane and normal line to a level surface $S: F(x, y, z) = k$ at the point (x_0, y_0, z_0)
  - Tangent plane:
    $F_x(x_0, y_0, z_0)(x - x_0) + F_y(x_0, y_0, z_0)(y - y_0) + F_z(x_0, y_0, z_0)(z - z_0) = 0$
  - Normal line: $\frac{x - x_0}{F_x(x_0, y_0, z_0)} = \frac{y - y_0}{F_y(x_0, y_0, z_0)} = \frac{z - z_0}{F_z(x_0, y_0, z_0)}$

Exercise:

- Find equations of (a) the tangent plane and (b) the normal line to the surface $x = y^2 + z^2 + 1$ at the point $(3, 1, -1)$.